1992 - AB1

Let f be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.

(a) On what intervals is f increasing?

(b) On what intervals is the graph of f concave upward?

(c) Write the equation of each horizontal tangent line to the graph of f.

1992 - AB2

2. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.

(a) Find the minimum acceleration of the particle.

(b) Find the total distance traveled by the particle.

(c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

1992 - AB3

3. Let f be the function given by $f(x) = \ln\left|\dfrac{x}{1+x^2}\right|$.

(a) Find the domain of f.

(b) Determine whether f is an even function, an odd function, or neither. Justify your conclusion.

(c) At what values of x does f have a relative maximum or a relative minimum? For each such x, use the first derivative test to determine whether $f(x)$ is a relative maximum or a relative minimum.

(d) Find the range of f.

1992-AB4, BC1

4. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\frac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

1992-AB5, BC2

5. Let f be the function given by $f(x) = e^{-x}$, and let g be the function given by $g(x) = kx$, where k is the nonzero constant such that the graph of f is tangent to the graph of g.

(a) Find the x-coordinate of the point of tangency and the value of k.

(b) Let R be the region enclosed by the y-axis and the graphs of f and g. Using the results found in part (a), determine the area of R.

(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated by revolving the region R, given in part (b), about the x-axis.

1992-AB6

6. At time t, $t \ge 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) Find the radius of the sphere as a function of t.

(b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

1992 - BC 3

3. At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the xy-plane is given by the arametric equations $x = e^t \sin t$ and $y = e^t \cos t$.

(a) Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

(b) Find the speed of the particle when $t = 1$.

(c) Find the distance traveled by the particle along the path from $t = 0$ to $t = 1$.

1992 - BC 4

4. Let f be a function defined by $f(x) = \begin{cases} 2x - x^2 & \text{for } x \le 1, \\ x^2 + kx + p & \text{for } x > 1. \end{cases}$

(a) For what values of k and p will f be continuous and differentiable at $x = 1$?

(b) For the values of k and p found in part (a), on what interval or intervals is f increasing?

(c) Using the values of k and p found in part (a), find all points of inflection of the graph of f. Support your conclusion.

1992 - BC 5

5 The length of a solid cylindrical cord of elastic material is 32 inches. A circular cross section of the cord has radius $\frac{1}{2}$ inch.

(a) What is the volume, in cubic inches, of the cord?

(b) The cord is stretched lengthwise at a constant rate of 18 inches per minute. Assuming that the cord maintains a cylindrical shape and a constant volume, at what rate is the radius of the cord changing one minute after the stretching begins? Indicate units of measure.

(c) A force of $2x$ pounds is required to stretch the cord x inches beyond its natural length of 32 inches. How much work is done during the first minute of stretching described in part (b)? Indicate units of measure.

1992 - BC 6

6. Consider the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln(n)}$, where $p \ge 0$.

(a) Show that the series converges for $p > 1$.

(b) Determine whether the series converges or diverges for $p = 1$. Show your analysis.

(c) Show that the series diverges for $0 \le p < 1$.